

A Simple Geometry of Income Elasticities

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MICHAEL WATTS, Section Editor

A Simple Geometry of Income Elasticities

Jerome F. Heavey

Although most students of economics become acquainted with the graphical analysis of the income and substitution effects of a price change, they often fail to appreciate that the same graph provides information on the income elasticities of the two goods. The standard diagram provides a quick and easy method for determining, by sight, whether each of the goods is inferior, normal, or superior.

Figure 1 contains the familiar Hicksian decomposition of the income and substitution effects for a decrease in the price of good X . The original budget line is KL , on which the consumer had selected the most preferred combination of goods at point A on indifference curve U_1 . Subsequent to the price decrease, the consumer will select the most preferred point, on budget line KM . The compensated budget line RS , drawn according to Hick's method, locates the point C on indifference curve U_1 .

Draw a ray from the origin through point C , and draw a vertical line through the same point C . Both the ray and the vertical line intersect budget line KM , and the ray and the vertical line are the boundaries for the three classes of good into which X might fall. Let the optimum combination of goods on budget line KM be denoted by the letter B .

1. If B is to the left of the vertical line, then X is an inferior good.
2. If B is between the vertical line and the ray, including the locations precisely on either the vertical line or the ray, then X is a normal good.
3. If point B is to the right of the ray, then X is a superior good.

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PROOF

Inferiority

Good X is inferior if the quantity of X decreases as income increases. Hence, when X is inferior, the optimal point on budget line KM will be to the left of the vertical line drawn through the optimal point on budget line RS . Good X would be a Giffen good if point B , the optimal point on the final budget line, was to the left of the vertical line drawn through point A , the optimal point on the original budget line.¹

Normality

Good X is normal (and not superior) if the quantity of X increases or remains the same as income increases, but the proportion of income devoted to the purchase of X does not increase. The points on budget line KM that lie within the region of normality are therefore those from the vertical line to the ray. At the intersection of the vertical line through point C and budget line KM , the quantity of X is the same as it had been on budget line RS ; hence, X is a normal good. To the right of the vertical line, the quantity of X will be greater on budget line KM than it had been on budget line RS , and so X will be a normal good because the quantity consumed has increased as income increased. It remains only to show that a location for point B to the right of the ray would constitute an increase in the *proportion* of income devoted to X , making X a superior good.

Superiority

The prices of X and Y , P_x and P_y , respectively, do not change as income changes. Let the quantity of X at point C be called X_1 , and let the quantity of X at point B be called X_2 . Money income is M_1 at point C and M_2 at point B , the two amounts of money differing by the compensating variation. Therefore, X is a superior good if and only if

$$\begin{array}{l} \text{Proportion of income} \\ \text{spent on good } X \\ \text{at point } B \end{array} > \begin{array}{l} \text{Proportion of income} \\ \text{spent on good } X \\ \text{at point } C. \end{array} \quad (1)$$

This relation can be expressed as

$$\frac{P_x X_2}{M_2} > \frac{P_x X_1}{M_1}. \quad (2)$$

The total amount of money spent at point C or at point B is the sum of the amounts spent on goods X and Y , so (2) may be rewritten as

$$\frac{P_x X_2}{P_x X_2 + P_y Y_2} > \frac{P_x X_1}{P_x X_1 + P_y Y_1}, \quad (3)$$

where Y_1 and Y_2 are the amounts of Y consumed at points C and B , respectively.

The equation for the ray through point C is $Y = b_1 X$, and the ray from the origin through point B would have the equation $Y = b_2 X$, where b_1 and b_2 are the slopes of the two lines. Hence, at point C , $Y_1 = b_1 X_1$, and at point B , $Y_2 = b_2 X_2$, so equation (3) is equivalent to

$$\frac{P_x X_2}{P_x X_2 + P_y b_2 X_2} > \frac{P_x X_1}{P_x X_1 + P_y b_1 X_1} \quad (4)$$

$$(P_x X_2) (P_x X_1 + P_y b_1 X_1) > (P_x X_1) (P_x X_2 + P_y b_2 X_2) \quad (5)$$

$$P_x X_2 P_x X_1 + P_x X_2 P_y b_1 X_1 > P_x X_1 P_x X_2 + P_x X_1 P_y b_2 X_2 \quad (6)$$

$$X_2 P_x X_1 + X_2 P_y b_1 X_1 > X_1 P_x X_2 + X_1 P_y b_2 X_2 \quad (7)$$

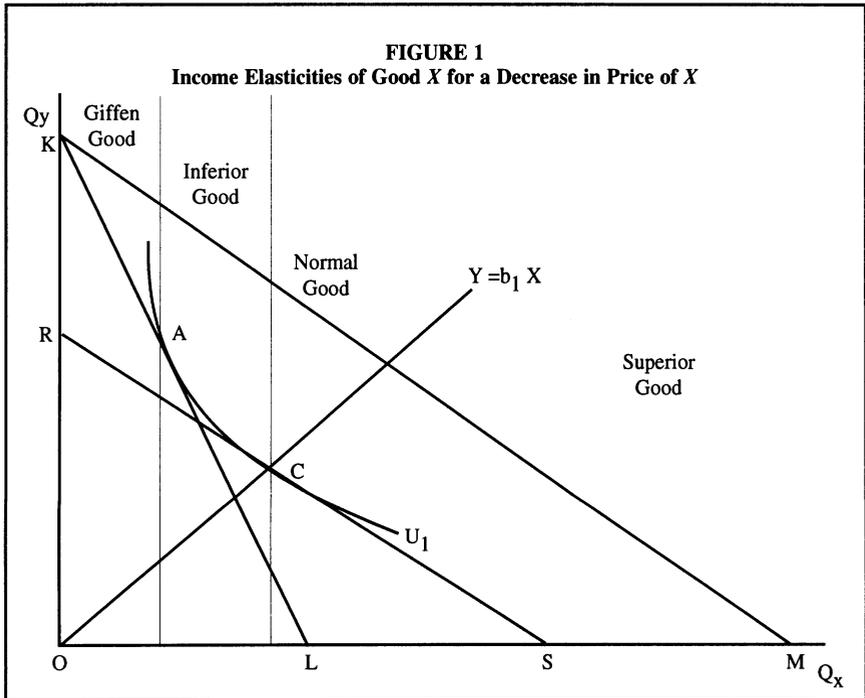
$$P_x X_1 + P_y b_1 X_1 > X_1 P_x + X_1 P_y b_2 \quad (8)$$

$$P_x + P_y b_1 > P_x + P_y b_2 \quad (9)$$

$$P_y b_1 > P_y b_2 \quad (10)$$

$$b_1 > b_2 \quad (11)$$

Thus, equation (1), which defines a superior good, is equivalent to equation (11), thereby demonstrating that X is a superior good if the ray drawn from the origin through the optimal point on the final budget line has a lesser slope than does the ray drawn from the origin through the optimal point on the compensated budget line.



Because we can observe the response of the Y quantity when income changes from the compensated budget line to the final budget line, we can readily classify the income elasticities for Y . Recall that “point B ” is the name given to the combination of goods that the consumer selects on the final budget line. Draw a horizontal line through point C on Figure 1. If point B lies below the horizontal line, then Y is an inferior good. If point B lies between the horizontal line and the ray through point C (including the points exactly on either of these boundaries), then Y is a normal good. If B lies above the ray, then Y is a superior good.² The proofs of these statements follow the logic shown for X .

The same logic allows a simple, visual classification of the income elasticities for each good in each of four cases: either an increase or a decrease in either the price of X or the price of Y . Figures 2, 3, 4, and 5 provide the complete taxonomy.

A compact notation is used in each of the tables and is particularly necessary for tables 3 and 5, where point B , the optimal combination of goods on the final budget line, will be located on the innermost budget line of the drawing. The notation employs the letters X and Y to indicate the two goods and the letters N , I , and S to indicate normality, inferiority, or superiority. For example, the legend $X = N$ indicates the portion of the final budget line along which X is a normal good, and the legend $Y = S$ indicates the portion of the final budget line along which Y is a superior good.

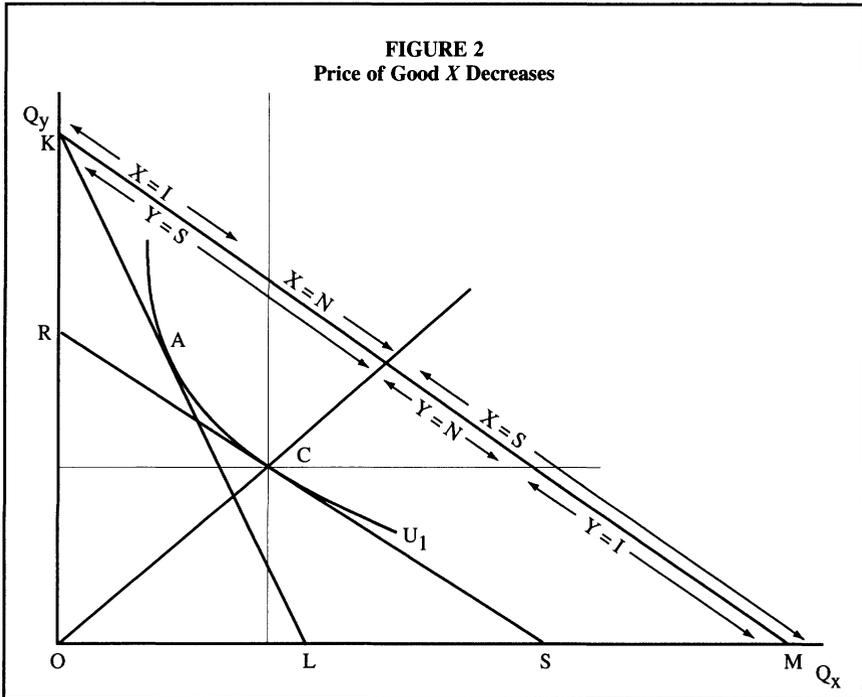


FIGURE 3
Price of Good X Increases

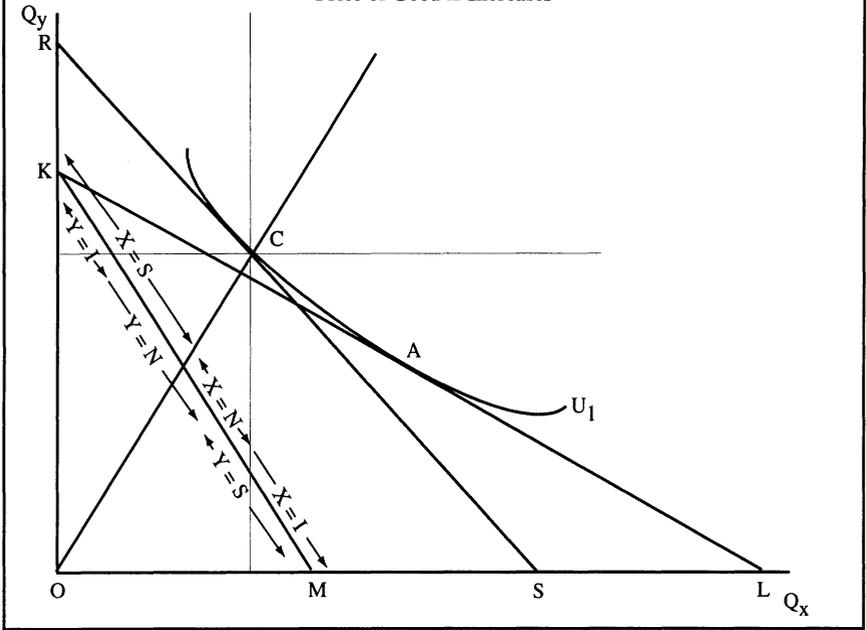
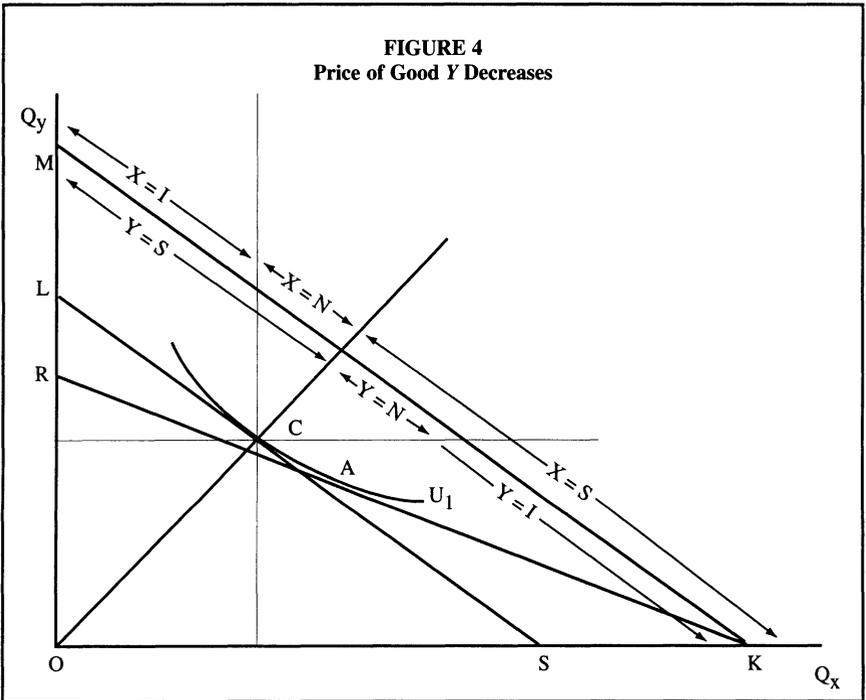
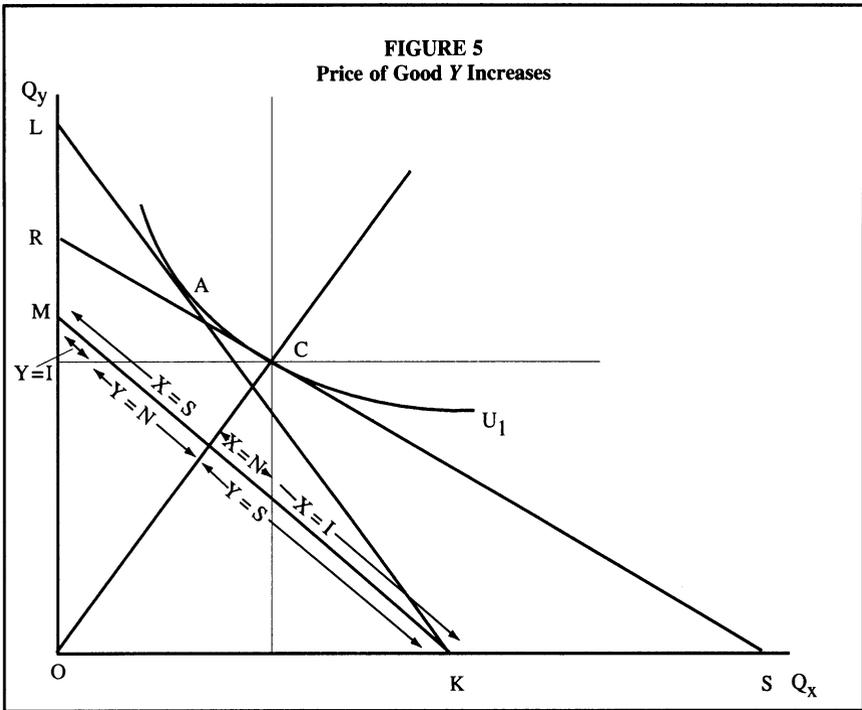


FIGURE 4
Price of Good Y Decreases





NOTES

1. Some economists, for example, Friedman (1990, 55), question whether Giffen goods even exist. Stigler (1987, 23) has tellingly noted that almost a century after Alfred Marshall defined the Giffen paradox, no one has yet demonstrated the existence of even one Giffen good. And this is perhaps the most important thing about Giffen goods, for their absence is an argument for the universality of the law of demand.
2. Notice that if one good is inferior, the other must be superior, but if one good is superior the other good may be either normal or inferior. Also, it does not seem likely to be a common occurrence that both goods would be merely normal, that is, both goods normal and neither good superior, for this would require the coincidence of point *B* lying precisely on the ray drawn through point *C*.

REFERENCES

- Friedman, D. D. 1990. *Price theory*. Cincinnati: South-Western.
- Stigler, G. J. 1987. *The theory of price*. 4th ed. New York: Macmillan.